Spiral cylindrique sans courbes terminales

Poids du spiral et anisochronisme en position verticale

Déformations planes

Caractéristiques du spiral ***dextre***

Référence :E:\Résonateur (TA)\Le spiral cylindrique\SC sans CT - Poids du spiral - Th_élémentaire.mcd(R)

Référence :E:\Résonateur (TA)\Data\Bal_spiral cylindrique (ex num).mcd(R)

Dimensions $\quad ép = 0.09\,mm \quad ha = 0.334\,mm \quad S = 0.03\,mm^2 \quad R_0 = 5\,mm \quad TOL := 10^{-9}$

Elinvar $\quad \rho_s = 8\times10^3\ kg\cdot m^{-3} \quad E = 1.7\times10^{11}\,Pa \quad m_s = 77.534\,mg$

Forme initiale du spiral

$r_0(\alpha) := R_0 \quad n_s := 10.15 \quad \psi_0 := n_s\cdot 360\cdot deg \quad \psi_0 = 3.654\times10^3\,deg \quad L := R_0\cdot\psi_0 \quad L = 318.872\,mm$

$x_0(\alpha) := R_0\cdot cos(\alpha) \quad y_0(\alpha) := R_0\cdot sin(\alpha) \quad s(\alpha) := R_0\cdot\alpha$

Position du piton $\quad r_P := R_0 \quad \alpha_P := 0 \quad x_P = 5\,mm \quad y_P = 0\,mm$

Position du point d'attache à la virole $\quad r_V := R_0 \quad \alpha_V(\theta) := \psi_0 + \theta \quad x_V(\theta) := r_V\cdot cos(\alpha_V(\theta)) \quad y_V(\theta) := r_V\cdot sin(\alpha_V(\theta))$

Amplitude stationnaire du balancier $\quad \theta_0 = 270\,deg$

Moment quadratique de section

Référence :E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$I_{33} := I_{f\_rect}(ép, ha)$

Théorie élémentaire (Defossez)

Déplacement du centre de masse $\quad \xi_{él}(\theta) := \xi_{él}(\theta, n_s) \quad \eta_{él}(\theta) := \eta_{él}(\theta, n_s) \quad \zeta_{él}(\theta) := \zeta_{él}(\theta, n_s)$

Perturbation de marche $\quad \mu_{él}(\theta_0) := \mu_{él}(\theta_0, n_s)$

Première approximation de la déformée du spiral

$$\varphi_0(\alpha) := \alpha + \frac{\pi}{2} \qquad z_P := x_P + i\cdot y_P \qquad z_1(\theta,\alpha) := z_P + R_0\cdot\int_0^{\alpha} i\cdot exp(i\cdot\alpha')\cdot exp\left(i\cdot\theta\cdot\frac{\alpha'}{\psi_0}\right)d\alpha'$$

$$z_0(\alpha) := R_0\cdot exp(i\cdot\alpha) \qquad z_1(\theta,\alpha) := z_0(\alpha)\cdot exp\left(i\cdot\theta\cdot\frac{\alpha}{\psi_0}\right) - \frac{i\cdot\theta}{\psi_0}\cdot\int_0^{\alpha} z_0(\alpha')\cdot exp\left(i\cdot\theta\cdot\frac{\alpha'}{\psi_0}\right)d\alpha'$$

$$z_1(\theta,\alpha) := z_P + R_0\cdot\frac{\psi_0}{\psi_0+\theta}\cdot\left(exp\left(i\cdot\alpha\cdot\frac{\psi_0+\theta}{\psi_0}\right) - 1\right)$$

Première approximation du déplacement du centre de gravité

$$\zeta_{1s}(\theta) := \frac{1}{L}\cdot\int_0^{\psi_0} z_1(\theta,\alpha)\cdot R_0\, d\alpha \qquad \zeta_{1s}(\theta_0) = 0.304 + 0.013i\, mm$$

Analytiquement

$$\mathbf{\Delta_1}(\theta) := \frac{i\cdot\theta}{\psi_0}\cdot\int_0^{\psi_0} z_0(\alpha)\cdot exp\left(i\cdot\theta\cdot\frac{\alpha}{\psi_0}\right) d\alpha \qquad \mathbf{\Delta_1}(\theta) := \frac{\theta}{\psi_0+\theta}\cdot R_0\cdot\left[exp\left[i\cdot(\psi_0+\theta)\right]-1\right]$$

$$u_1(\theta) := Re\left(\mathbf{\Delta_1}(\theta)\right) \qquad v_1(\theta) := Im\left(\mathbf{\Delta_1}(\theta)\right) \qquad u_1(\theta_0) = -0.066\, mm \qquad v_1(\theta_0) = -0.202\, mm$$

$$\xi_{1s}(\theta) := \frac{d}{d\theta}v_1(\theta) - u_1(\theta) \qquad \xi_{1s}(\theta) := \frac{\theta\cdot R_0}{\psi_0+\theta} + \frac{\psi_0}{(\psi_0+\theta)^2}\cdot R_0\cdot sin(\psi_0+\theta) \qquad \xi_{1s}(\theta_0) = 0.304\, mm$$

$$\eta_{1s}(\theta) := \frac{d}{d\theta}u_1(\theta) - v_1(\theta) \qquad \eta_{1s}(\theta) := \frac{\psi_0}{(\psi_0+\theta)^2}\cdot\left[R_0\cdot\left(1-cos(\psi_0+\theta)\right)\right] \qquad \eta_{1s}(\theta_0) = 0.013\, mm$$

Calcul des réactions

$$\xi_{0s} := \frac{R_0}{\psi_0}\cdot sin(\psi_0) \qquad \eta_{0s} := \frac{R_0}{\psi_0}\cdot\left(1-cos(\psi_0)\right)$$

$$q2_{0s} := \frac{R_0^{\,2}}{2\cdot\psi_0}\cdot\left(\psi_0 - cos(\psi_0)\cdot sin(\psi_0)\right) \qquad p2_{0s} := \frac{R_0^{\,2}}{2\cdot\psi_0}\cdot\left(\psi_0 + cos(\psi_0)\cdot sin(\psi_0)\right) \qquad k_{0s} := \frac{R_0^{\,2}}{2\cdot\psi_0}\cdot sin(\psi_0)^2$$

$$\xi_{0s} = 0.063\, mm \qquad \eta_{0s} = 0.032\, mm \qquad q2_{0s} = 12.407\, mm^2 \qquad p2_{0s} = 12.593\, mm^2 \qquad k_{0s} = 0.128\, mm^2$$

$$\mathbf{S_0} := \frac{L}{E\cdot I_{33}}\cdot\begin{pmatrix} q2_{0s} - \eta_{0s}^{\,2} & \eta_{0s}\cdot\xi_{0s} - k_{0s} \\ \eta_{0s}\cdot\xi_{0s} - k_{0s} & p2_{0s} - \xi_{0s}^{\,2} \end{pmatrix} \qquad \mathbf{R'}(\theta) := \mathbf{S_0}^{-1}\cdot\begin{pmatrix} u_1(\theta) \\ v_1(\theta) \end{pmatrix} \qquad \mathbf{R'}(\theta_0) = \begin{pmatrix} -5.914\times 10^{-5} \\ -1.746\times 10^{-4} \end{pmatrix} N$$

$$\left|\mathbf{R'}(\theta_0)\right| = 1.843\times 10^{-4}\, N$$

Approximations

$$\sigma 2 := \frac{1}{\psi_0}\cdot\int_0^{\psi_0} z_0(\alpha)\cdot\overline{z_0(\alpha)}\, d\alpha \qquad \sigma 2 := R_0^{\,2} \qquad \frac{\sigma 2}{2} = 12.5\, mm^2$$

$$R'(\theta) := \frac{E\cdot I_{33}}{L}\cdot\frac{2}{\sigma 2}\cdot\begin{pmatrix} u_1(\theta) \\ v_1(\theta) \end{pmatrix} \qquad R'(\theta_0) = \begin{pmatrix} -5.693\times 10^{-5} \\ -1.752\times 10^{-4} \end{pmatrix} N \qquad \left|R'(\theta_0)\right| = 1.842\times 10^{-4}\, N$$

Deuxième approximation du déplacement du centre de gravité

$$x_1(\theta,\alpha) := Re\left(z_1(\theta,\alpha)\right) \qquad y_1(\theta,\alpha) := Im\left(z_1(\theta,\alpha)\right)$$

$$s\xi_1(\theta,\alpha) := \int_0^{\alpha} x_1(\theta,\alpha')\cdot R_0\, d\alpha' \qquad \xi_{1s}(\theta) := \frac{1}{L}\cdot s\xi_1(\theta,\psi_0) \qquad \xi_{1s}(\theta_0) = 0.304\, mm$$

$$s\eta_1(\theta,\alpha) := \int_0^{\alpha} y_1(\theta,\alpha')\cdot R_0\, d\alpha' \qquad \eta_{1s}(\theta) := \frac{1}{L}\cdot s\eta_1(\theta,\psi_0) \qquad \eta_{1s}(\theta_0) = 0.013\, mm$$

$$sp2_1(\theta,\alpha) := \int_0^{\alpha} x_1(\theta,\alpha')^2 \cdot R_0 \, d\alpha' \qquad p2_1(\theta) := \frac{1}{L} \cdot sp2_1(\theta,\psi_0) \qquad p2_1(\theta_0) = 10.855\, mm^2$$

$$sq2_1(\theta,\alpha) := \int_0^{\alpha} y_1(\theta,\alpha')^2 \cdot R_0 \, d\alpha' \qquad q2_1(\theta) := \frac{1}{L} \cdot sq2_1(\theta,\psi_0) \qquad q2_1(\theta_0) = 10.914\, mm^2$$

$$sk_1(\theta,\alpha) := \int_0^{\alpha} x_1(\theta,\alpha') \cdot y_1(\theta,\alpha') \cdot R_0 \, d\alpha' \qquad k_1(\theta) := \frac{1}{L} \cdot sk_1(\theta,\psi_0) \qquad k_1(\theta_0) = 0.059\, mm^2$$

$$\boldsymbol{S_1}(\theta,\alpha) := \frac{1}{E \cdot I_{33}} \cdot \begin{pmatrix} -y_1(\theta,\alpha) \cdot s\eta_1(\theta,\alpha) + sq2_1(\theta,\alpha) & y_1(\theta,\alpha) \cdot s\xi_1(\theta,\alpha) - sk_1(\theta,\alpha) \\ x_1(\theta,\alpha) \cdot s\eta_1(\theta,\alpha) - sk_1(\theta,\alpha) & -x_1(\theta,\alpha) \cdot s\xi_1(\theta,\alpha) + sp2_1(\theta,\alpha) \end{pmatrix}$$

$$\boldsymbol{R'}(\theta) := \boldsymbol{S_1}(\theta,\psi_0)^{-1} \cdot \begin{pmatrix} x_V(\theta) - x_1(\theta,\psi_0) \\ y_V(\theta) - y_1(\theta,\psi_0) \end{pmatrix} \qquad \boldsymbol{R'}(\theta_0) = \begin{pmatrix} -8.356 \times 10^{-5} \\ -2.281 \times 10^{-4} \end{pmatrix} N$$

$$R'_x(\theta) := \boldsymbol{R'}(\theta)_0 \qquad R'_y(\theta) := \boldsymbol{R'}(\theta)_1$$

$$\xi_{2s}(\theta) := \xi_{1s}(\theta) + \frac{1}{\psi_0} \cdot \left(R'_x(\theta) \cdot \int_0^{\psi_0} \boldsymbol{S_1}(\theta,\alpha)_{0,0} \, d\alpha + R'_y(\theta) \cdot \int_0^{\psi_0} \boldsymbol{S_1}(\theta,\alpha)_{0,1} \, d\alpha \right) \qquad \xi_{2s}(\theta_0) = 0.261\, mm$$

$$\eta_{2s}(\theta) := \eta_{1s}(\theta) + \frac{1}{\psi_0} \cdot \left(R'_x(\theta) \cdot \int_0^{\psi_0} \boldsymbol{S_1}(\theta,\alpha)_{1,0} \, d\alpha + R'_y(\theta) \cdot \int_0^{\psi_0} \boldsymbol{S_1}(\theta,\alpha)_{1,1} \, d\alpha \right) \qquad \eta_{2s}(\theta_0) = -0.1\, mm$$

Approximations de Haag

Vérification des approximations

$$\sigma 2_1(\theta) := \frac{1}{\psi_0} \cdot \int_0^{\psi_0} z_1(\theta,\alpha) \cdot \overline{z_1(\theta,\alpha)} \, d\alpha \qquad \frac{\sigma 2_1(\theta_0)}{2} = 10.884\, mm^2$$

$$q2_{1s}(\theta) := \frac{1}{L} \cdot \int_0^{\psi_0} y_1(\theta,\alpha)^2 \cdot R_0 \, d\alpha \qquad p2_{1s}(\theta) := \frac{1}{L} \cdot \int_0^{\psi_0} x_1(\theta,\alpha)^2 \cdot R_0 \, d\alpha$$

$$k_{1s}(\theta) := \frac{1}{L} \cdot \int_0^{\psi_0} x_1(\theta,\alpha) \cdot y_1(\theta,\alpha) \cdot R_0 \, d\alpha$$

$$q2_{1s}(\theta_0) = 10.914\, mm^2 \qquad p2_{1s}(\theta_0) = 10.855\, mm^2 \qquad k_{1s}(\theta_0) = 0.059\, mm^2$$

$$\Sigma 2_1(\theta) := \frac{1}{\psi_0^2} \cdot \int_0^{\psi_0} \alpha \cdot z_1(\theta,\alpha) \cdot \overline{z_1(\theta,\alpha)} \, d\alpha \qquad \frac{\Sigma 2_1(\theta_0)}{2} = 5.435\, mm^2$$

$$Q2_{1s}(\theta) := \frac{1}{L^2} \cdot \int_0^{\psi_0} R_0 \cdot \alpha \cdot y_1(\theta,\alpha)^2 \cdot R_0 \, d\alpha \qquad P2_{1s}(\theta) := \frac{1}{L^2} \cdot \int_0^{\psi_0} R_0 \cdot \alpha \cdot x_1(\theta,\alpha)^2 \cdot R_0 \, d\alpha$$

$$Q2_{1s}(\theta_0) = 5.495\, mm^2 \qquad P2_{1s}(\theta_0) = 5.375\, mm^2$$

$$\xi_{as}(\theta) := \frac{d}{d\theta}v_1(\theta) - \frac{\Sigma 2_1(\theta)}{\sigma 2_1(\theta)}\cdot u_1(\theta) - \frac{v_1(\theta)}{2\cdot\sigma 2_1(\theta)}\cdot\frac{d}{d\theta}\sigma 2_1(\theta) \qquad \xi_{as}(\theta_0) = 0.269\,mm$$

$$\eta_{as}(\theta) := -\frac{d}{d\theta}u_1(\theta) - \frac{\Sigma 2_1(\theta)}{\sigma 2_1(\theta)}\cdot v_1(\theta) + \frac{u_1(\theta)}{2\cdot\sigma 2_1(\theta)}\cdot\frac{d}{d\theta}\sigma 2_1(\theta) \qquad \eta_{as}(\theta_0) = -0.087\,mm$$

$$\kappa := \frac{1}{\sigma 2\cdot L^2}\cdot\int_0^{\psi_0} R_0\cdot\alpha\cdot z_0(\alpha)\cdot\overline{z_0(\alpha)}\cdot R_0\,d\alpha \qquad \kappa = 0.5$$

$$\xi_s(\theta) := \frac{d}{d\theta}v_1(\theta) - \kappa\cdot u_1(\theta) \qquad \eta_s(\theta) := -\frac{d}{d\theta}u_1(\theta) - \kappa\cdot v_1(\theta) \qquad \xi_s(\theta_0) = 0.271\,mm$$

$$\eta_s(\theta_0) = -0.088\,mm$$

$$\zeta(\theta) := -i\cdot\frac{d}{d\theta}\mathbf{\Delta 1}(\theta) - \kappa\cdot\mathbf{\Delta 1}(\theta) \qquad \zeta(\theta_0) = 0.271 - 0.088i\,mm$$

$$\zeta(\theta) := \frac{R_0}{L}\cdot\int_0^{\psi_0} e^{i\cdot\alpha}\cdot e^{i\cdot\theta\cdot\frac{\alpha}{\psi_0}}\cdot\left[1 + i\cdot\theta\cdot\left(\frac{\alpha}{\psi_0} - \kappa\right)\right]\cdot R_0\,d\alpha \qquad \zeta(\theta_0) = 0.271 - 0.088i\,mm$$

$$\zeta(\theta) := \frac{R_0\cdot\psi_0}{(\psi_0+\theta)^2}\cdot\left[\left(i + \theta\cdot\kappa + \frac{\theta^2}{\psi_0}\cdot\kappa\right) - \left[i - \left(\theta + \frac{\theta^2}{\psi_0}\right)\cdot(1-\kappa)\right]\cdot exp\left[i\cdot(\psi_0+\theta)\right]\right]$$

$$\zeta(\theta_0) = 0.271 - 0.088i\,mm$$

Formule de Haag

$$\zeta_{ah}(\theta) := \frac{R_0}{\psi_0}\cdot i\cdot\left[1 - i\cdot\frac{\theta}{2} - \left(1 + i\cdot\frac{\theta}{2}\right)\cdot e^{i\cdot(\psi_0+\theta)}\right] \qquad \omega(\theta) := \frac{\psi_0+\theta}{2}$$

$$\zeta_{ah}(\theta) := \frac{R_0}{\psi_0}\cdot e^{i\cdot\omega(\theta)}\cdot(2\cdot sin(\omega(\theta)) + \theta\cdot cos(\omega(\theta))) \qquad \zeta_{ah}(\theta_0) = 0.288 - 0.094i\,mm$$

Graphes du déplacement du centre de gravité

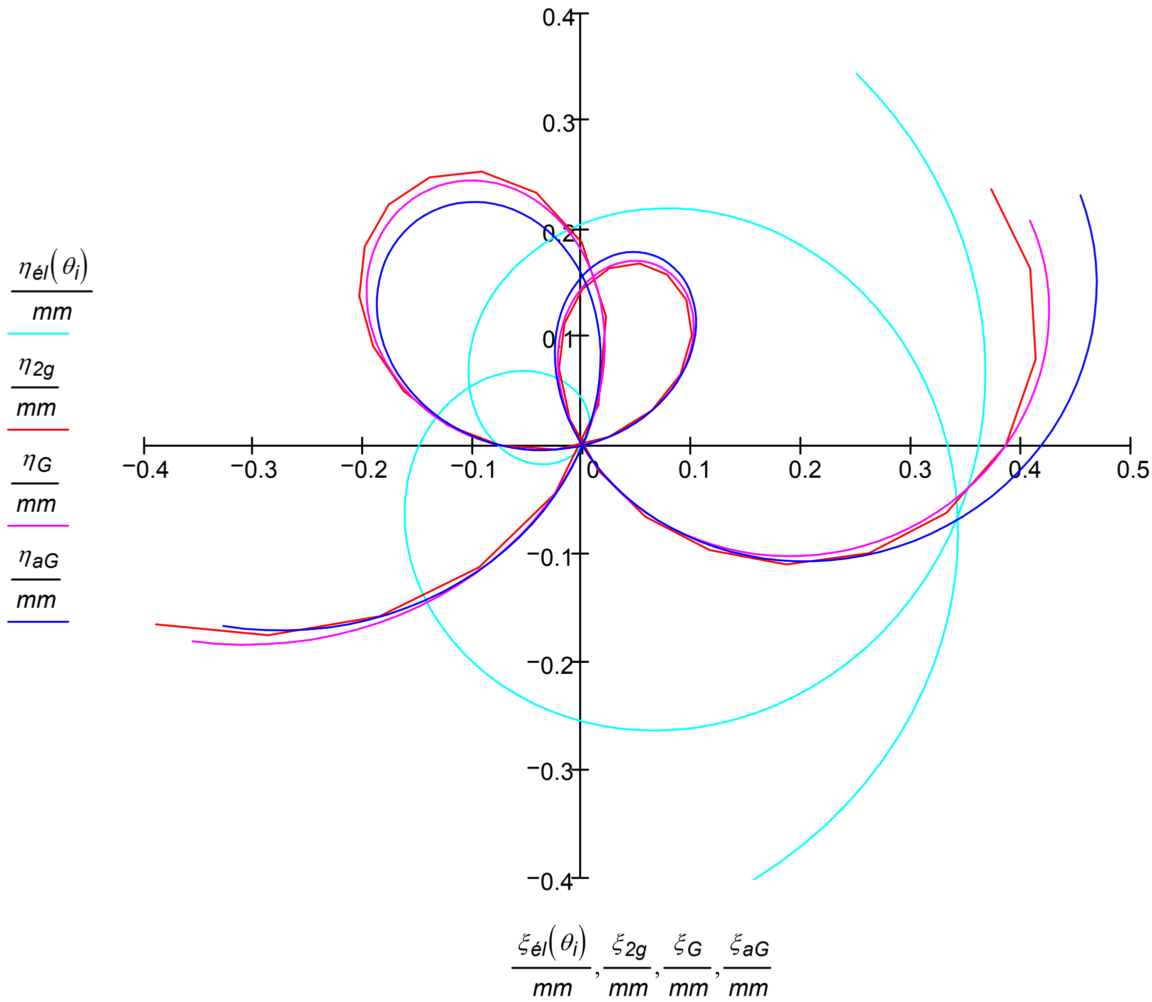
$$n := 201 \qquad i := 0..n-1 \qquad \Delta\theta := \frac{4\cdot\pi}{n-1} \qquad \theta_i := -2\cdot\pi + i\cdot\Delta\theta$$

$$m := 41 \qquad j := 0..m-1 \qquad \Delta\theta_m := \frac{4\cdot\pi}{m-1} \qquad \theta_{m_j} := -2\cdot\pi + j\cdot\Delta\theta_m$$

$$\xi_{2g_j} := \xi_{2s}(\theta_{m_j}) \qquad \eta_{2g_j} := \eta_{2s}(\theta_{m_j}) \qquad \xi_{2g_i} := 0 \qquad \eta_{2g_i} := 0$$

$$\xi_{G_i} := Re(\zeta(\theta_i)) \qquad \eta_{G_i} := Im(\zeta(\theta_i))$$

$$\xi_{aG_i} := Re(\zeta_{ah}(\theta_i)) \qquad \eta_{aG_i} := Im(\zeta_{ah}(\theta_i))$$



Perturbation de période - spiral non déformé en position de repos

Calcul par intégrations numérqiues

$$\eta(\theta) := Im(\zeta(\theta)) \qquad \Gamma(\theta) := -m_s \cdot g \cdot \frac{d}{d\theta}\eta(\theta)$$

$$\theta(\varphi) := \theta_0 \cdot cos(\varphi) \qquad \Delta(\theta_0) := \frac{L}{2 \cdot \pi \cdot \theta_0 \cdot E \cdot I_{33}} \cdot \int_0^{2 \cdot \pi} \Gamma(\theta_0 \cdot cos(\varphi)) \cdot cos(\varphi)\, d\varphi \qquad \Delta(\theta_0) = 6.243 \times 10^{-4}$$

$$\zeta'(\theta) := \frac{d}{d\theta}\zeta(\theta) \qquad \zeta'(\theta_0) = 0.219 + 0.079i\, mm$$

$$\zeta'(\theta) := \frac{R_0}{L} \cdot \int_0^{\psi_0} z_0(\alpha) \cdot exp\left(i \cdot \theta \cdot \frac{s(\alpha)}{L}\right) \cdot \left[i \cdot \left(2 \cdot \frac{s(\alpha)}{L} - \kappa\right) - \theta \cdot \frac{s(\alpha)}{L} \cdot \left(\frac{s(\alpha)}{L} - \kappa\right)\right] d\alpha$$

$$f(\theta_0, \alpha) := \int_0^{2 \cdot \pi} exp\left(i \cdot \theta_0 \cdot \frac{\alpha}{\psi_0} \cdot cos(\varphi)\right) \cdot \left[i \cdot \left(2 \cdot \frac{\alpha}{\psi_0} - \kappa\right) - \theta_0 \cdot \frac{\alpha}{\psi_0} \cdot \left(\frac{\alpha}{\psi_0} - \kappa\right) \cdot cos(\varphi)\right] \cdot cos(\varphi)\, d\varphi$$

$$f(\theta_0, \alpha) := 2 \cdot \pi \cdot \left(\kappa - 2 \cdot \frac{\alpha}{\psi_0}\right) \cdot J1\left(\theta_0 \cdot \frac{\alpha}{\psi_0}\right) + \pi \cdot \left(\kappa - \frac{\alpha}{\psi_0}\right) \cdot \theta_0 \cdot \frac{\alpha}{\psi_0} \cdot \left(J0\left(\theta_0 \cdot \frac{\alpha}{\psi_0}\right) - Jn\left(2, \theta_0 \cdot \frac{\alpha}{\psi_0}\right)\right)$$

$$f(\theta_0,\alpha) := 2\cdot\pi\cdot\frac{\alpha}{\psi_0}\cdot\left[\left(\kappa-\frac{\alpha}{\psi_0}\right)\cdot\theta_0\cdot J0\left(\theta_0\cdot\frac{\alpha}{\psi_0}\right)-J1\left(\theta_0\cdot\frac{\alpha}{\psi_0}\right)\right]$$

$$Z(\theta_0) := \frac{R_0}{2\cdot\pi\cdot\theta_0\cdot L}\cdot\int_0^{\psi_0} z_0(\alpha)\cdot f(\theta_0,\alpha)\,d\alpha \qquad Z(\theta_0) = 0.012 - 8.893i\times 10^{-3}\,mm$$

$$Z(\theta_0) := \frac{1}{\psi_0^{\,2}}\cdot\int_0^{\psi_0} z_0(\alpha)\cdot\alpha\cdot\left[\left(\kappa-\frac{\alpha}{\psi_0}\right)\cdot J0\left(\theta_0\cdot\frac{\alpha}{\psi_0}\right)-\frac{1}{\theta_0}\cdot J1\left(\theta_0\cdot\frac{\alpha}{\psi_0}\right)\right]d\alpha$$

$$Delta(\theta_0) := -g\cdot\frac{m_s\cdot L}{E\cdot I_{33}}\cdot Im(Z(\theta_0)) \qquad Delta(\theta_0) = 6.243\times 10^{-4}$$

$$\mu(\theta_0) := -86400\cdot Delta(\theta_0) \qquad \boxed{\mu(\theta_0) = -53.942} \qquad \boxed{\mu(180\cdot deg) = -9.359}$$

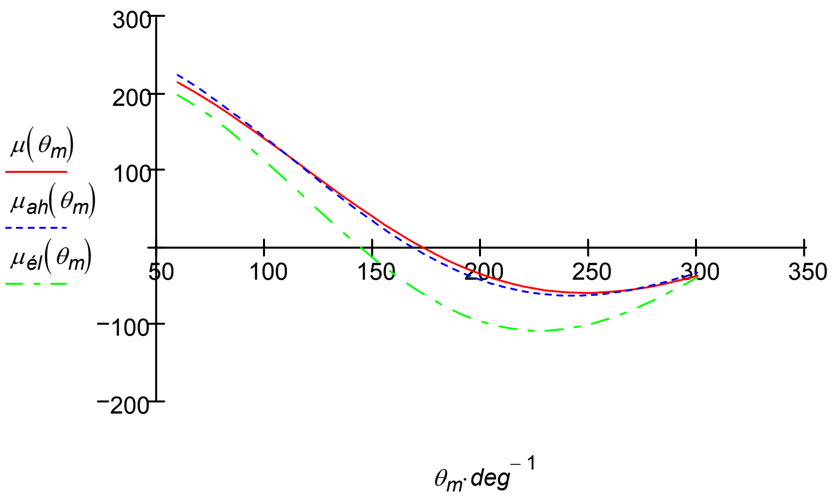
Approximation de Haag

$$z_{V0} := x_V(0) + i\cdot y_V(0) \qquad z_{V0} = 2.939 + 4.045i\,mm$$

$$P(\theta_0) := \frac{1}{2}\cdot J0(\theta_0) + \frac{1}{\theta_0}\cdot J1(\theta_0) \qquad Z_{ah}(\theta_0) := \frac{i}{\psi_0}\cdot z_{V0}\cdot P(\theta_0) \qquad \delta_{ah}(\theta_0) := -g\cdot\frac{m_s\cdot L}{E\cdot I_{33}}\cdot Im(Z_{ah}(\theta_0))$$

$$\mu_{ah}(\theta_0) := -86400\cdot\delta_{ah}(\theta_0) \qquad \boxed{\mu_{ah}(\theta_0) = -53.862} \qquad \boxed{\mu_{ah}(180\cdot deg) = -17.197}$$

$$\theta_m := 60\cdot deg, 65\cdot deg .. 300\cdot deg$$



$$\theta_1 := 170\cdot deg \qquad \theta_1 := racine(P(\theta_1), \theta_1) \qquad \theta_1 = 169\,deg$$

$$\theta_2 := 320\cdot deg \qquad \theta_2 := racine(P(\theta_2), \theta_2) \qquad \theta_2 = 334.672\,deg$$